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A New Method For Solving A Fuzzy Solid Transportation Model With Fuzzy Ranking

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In this paper, the Vogel's approximation method (VAM) with imprecise parameters is performed in a solid transportation problem (STP). Here, for the first time, fuzzy ranking tool is introduced for the different type of operations, comparisons presence in VAM. Convert the obtained basic feasible solution (BFS) into an optimal solution by the method fuzzy modi-indices. Finally a numerical example is taken into account to support the method in imprecise environment. More-over, sensitivity analysis along with bounded technique is given.

Keywords: Solid Transportation Problem, Fuzzy Rank, VAM, Fuzzy MODI indices, Bounded Technique.

I. INTRODUCTION

In real-life, ranking of a fuzzy number is an important criteria in many decision making problem. Since Jain [1976][1978] employed the concept of maximizing set to order the fuzzy numbers in 1976, many authors have investigated various ranking methods. Some of these ranking methods have been compared and reviewed by Bortolan and Degani [1985] and more recently by Chen and Hwang [1972] and other contributions in this field by including an index for ordering fuzzy numbers defined by Choobineh and Li [1997]. Dias [1993] studied the method of ranking an alternative fuzzy number. Requena et. al [1994] proposed the ranking of fuzzy numbers using artificial neural networks. Ranking fuzzy values with satisfaction function investigated by Lee et. al [1998]. Ranking and defuzzification methods based on area compensation presented by Fortemps and Roubens [1996] and ranking alternatives with fuzzy weights using maximizing set and minimizing set given by Raj and Kumar [1999]. However, some these methods are counter-intuitive and difficult to implement, and others are counterintuitive and not discriminating. Furthermore, many of them produce different ranking outcomes for the same problem. In 1998, Lee and Li [1998] proposed a comparison of fuzzy numbers by considering the mean and dispersion (standard deviation) based on the uniform and the proportional probability distributions. Patra and Mondal [2012] discuss a new approach of ranking of generalized trapezoidal fuzzy numbers.Jain et. al[2016] and kin et. al [2017] proposed interval-based fuzzy ranking approach. Recently,Gu and Xuan[2017] discuss the ranking procedure of a fuzzy numbers based on possibility theory.

The Transportation Problem (TP), originally developed by Hitchcock[1941], is one of the most common combinatorial problems involving constraints that has been studied. In the TP bounds are given on two parameters availability of the sources and demand on the destination. The solid transportation problem (STP) may be considered as a special case of linear programming problem. In STP, bounds are given on three parameter namely, availability of source, demand of the destination and capacity of the conveyance. The STP was proposed by Schell [1955]. Haley [1962] introduced the solution procedure of STP which is an extension of the modified distribution method. For finding an optimal solution, the STP requires (m + n + l - 2) nonzero values of the decision variables to start with a basic feasible solution. Patel and Tripathy [1989] developed a computationally superior method for a STP with mixed constraints. In the real life the different type of cost is not certain enough, but varies with in a range, of uncertainty in the transportation cost considered by several researchers (cf. Ojha et. al.[2010], Samanta et. al.[2015]). Bit et. al. [1993] developed the fuzzy programming model for a multi-objective STP. Vajda [1988] proposed an algorithm for a multi-index transportation problem which is an extension of the modified distribution method. Basu et. al. [1994] provided an algorithm for finding the optimum solution of a solid fixed charge transportation problem. Gen et al. [1995] apply the genetic algorithm process for solving a bicriteria STP with fuzzy numbers. Li et. al. [1997] designed a neural network approach for a multicriteria STP. Li et. al.[1997] improved the genetic algorithm for solve the fuzzy multi-objective STP with fuzzy cost coefficient. Jimenez and Verdegay [1996,1998,1999] investigated STP in several forms. Pandian and Natarajan [2010] have introduced the zero point method for finding an optimal solution to a classical transportation problem. Recently Lixing et. al.[2007][2007][2014][2015] derived solid transportation problem with deferent type of fuzzy numbers. Vogel's approximation method(VAM) is one of the oldest method for the solution of transportation problem. In 2011,Vasko and Nelya discuss its really and simplicity. Samuel and Venkatachalapathy[2011] applied it for a fuzzy transportation problem with weighted mean. ,"Modified Vogel's Approximation Method Storozhyshina,"Balancing a transportation problem. Juman and Hoque Modified Vogel's approximation method for an unbalanced transportation problem. Recently, in 2015 Almaatani et. al [2015] proposed a Modified VAM for soling transportation problem.

In this paper, a solid transportation problem is considered with imprecise cost. The proposed STP is solved using modified vogal's approximation method. The obtained basic feasible solutions are converted into an optimal one by MODI method. The required operations and comparisons between the fuzzy numbers are execute by ranking method. Finally, explain the proposed method with an example. More over the result obtained through this method is compared with the result obtained by Lingo-9.0.

II. FUZZY NUMBER AND ITS RANK

Fuzzy Number: A fuzzy subset $\widetilde{A}$ of real number $\mathbf{R}$ with membership function $\mu_{\widetilde{A}} : \mathbf{R} \to [0,1]$ is called a fuzzy number if

(a) $\widetilde{A}$ is normal i.e. there exist an element x_0 such that $\mu_{\widetilde{A}}(x_0) = 1$;

(b) $\widetilde{A}$ is convex, i.e. $\mu_{\widetilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \mu_{\widetilde{A}}(x_1) \wedge \mu_{\widetilde{A}}(x_2)$ for all $x_1, x_2 \in \mathbf{R}$ and $\lambda \in [0,1]$;

(c) $\mu_{\widetilde{A}}$ is upper semi-continuous, and

(d) $supp(\widetilde{A})$ is bounded, here $supp(\widetilde{A}) = cl\{x \in \mathbf{R} : \mu_{\widetilde{A}}(x) > 0\}$, and cl is the closer operator.

Triangular Fuzzy Number (TFN): A triangular fuzzy number $\widetilde{A}$, with continuous membership function $\mu_{\widetilde{A}}(x) : X \to [0,1]$ which can be specified by the triplet (a_1, a_2, a_3) is as follows:

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0 & \text{if } x \leq a_1 \\ \dfrac{x-a_1}{a_2-a_1} & \text{if } a_1 \leq x \leq a_2 \\ \dfrac{a_3-x}{a_3-a_2} & \text{if } a_2 \leq x \leq a_3 \\ 0 & \text{if } x \geq a_3 \end{cases}$$

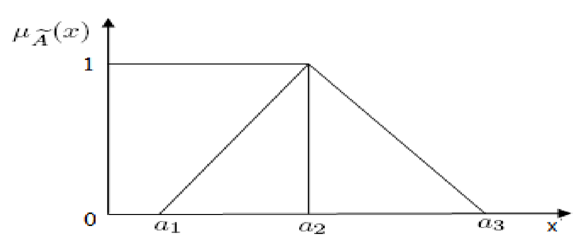

Figure 1. Membership function of TFN $\tilde{A}$

Trapezoidal Fuzzy Number (TrFN): TrFN is the fuzzy number with the membership function $\mu_{\widetilde{A}}(x)$, a continuous mapping $\mu_{\tilde{A}}(x) : \mathbf{R} \rightarrow [0,1]$

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1} & \text{for } a_1 \leq x \leq a_2 \\ 1 & \text{for } a_2 \leq x \leq a_3 \\ \dfrac{a_4-x}{a_4-a_3} & \text{for } a_3 \leq x \leq a_4 \\ 0 & \text{otherwise} \end{cases}$$

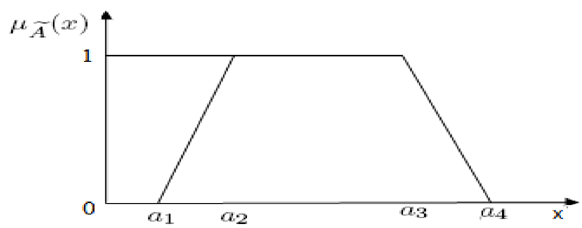

Figure 2. Membership function of TrFN $\tilde{A}$

Zadeh's Extension Principle: One of the basic concepts of fuzzy set theory which is used to generalize crisp mathematical concepts to fuzzy sets is the extension principle. Let X and Y be two universes and $f : X \longrightarrow Y$ be a crisp function. The extension principle tells us how to induce a mapping $f : P(X) \longrightarrow P(Y)$, where $P(X)$ and $P(Y)$ are the power sets of X and Y respectively.

The fuzzy extension principle is as follows:

We have the mapping $f : X \longrightarrow Y$, $y = f(x)$ which induce a function $f : \widetilde{A} \longrightarrow \widetilde{B}$ such that $\widetilde{B} = f(\widetilde{A}) = \{((y, \mu_{\tilde{B}}(y))|y = f(x), x \in X)\}$, where

$$\mu_{\tilde{B}}(y) = \begin{cases} \underset{x \in f^{-1}(y)}{sup} \quad \mu_{\tilde{A}}(x) & \text{if } f^{-1}(y) \neq \Phi, \\ 0 & \text{otherwise} \end{cases}$$

In general if f : $X_1 \times X_2 \times \ldots\ldots \times X_n \longrightarrow$ Y, and $\widetilde{A_1}, \widetilde{A_2}, \ldots., \widetilde{A_n}$ be a fuzzy sets in $X_1, X_2, \ldots\ldots, X_n$, the extension principle is defined as f : $\widetilde{A} \longrightarrow \widetilde{B}$ such that

$\widetilde{B} = \{((y, \mu_{\tilde{B}}(y))|y = f(x_1, x_2, \ldots, x_n), (x_1, x_2, \ldots, x_n) \in X)\}$, where

$$\mu_{\tilde{B}}(y) = \begin{cases} \underset{(x_1,x_2,\ldots,x_n) \in f^{-1}(y)}{sup} \quad min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2), \ldots., \mu_{\tilde{A}}(x_n)\} & \text{if } f^{-1}(y) \neq \Phi, \\ 0 & \text{otherwise} \end{cases}$$

α - Cut of a fuzzy number: An α- cut of a fuzzy number $\tilde{A}$ is defined as crisp set

$$A_\alpha = \{x : \mu_{\tilde{A}}(x) \geq \alpha, x \in X\} \text{ where } \alpha \in [0,1]$$

A_α is a non-empty bounded closed interval contained in X and it can be denoted by $A_\alpha = [A^L(\alpha), A^R(\alpha)]$. $A^L(\alpha)$ and $A^R(\alpha)$ are the lower and upper bounds of the closed interval respectively. A fuzzy number $\tilde{A}$ with α_1, α_2 - cut $A_{\alpha_1} = [A^L(\alpha_1), A^R(\alpha_1)], A_{\alpha_2} = [A^L(\alpha_2), A^R(\alpha_2)]$ and if $\alpha_2 \geq \alpha_1$, then $A^L(\alpha_2) \geq A^L(\alpha_1)$ and $A^R(\alpha_1) \geq A^R(\alpha_2)$.

Arithmetic of Fuzzy Numbers:

The fuzzy arithmetical operations under Extension Principle two trapezoidal fuzzy numbers $\widetilde{A} = (a_1, a_2, a_3, a_4)$ and $\widetilde{B} = (b_1, b_2, b_3, b_4)$ are

(i) The addition of $\widetilde{A}$ and $\widetilde{B}$ is $\widetilde{A} \oplus \widetilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$, where $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ and b_4 are any real numbers.

(ii) $-\widetilde{B} = (-b_4, -b_3, -b_2, -b_1)$, then the difference of $\widetilde{B}$ from $\widetilde{A}$ is
$\widetilde{A} \ominus \widetilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$, where $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ and b_4 are any real numbers.

(iii) The multiplication of $\widetilde{A}$ and $\widetilde{B}$ is $\widetilde{A} \otimes \widetilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4)$, where $\widetilde{A} \otimes \widetilde{B}$ is a trapezoidal fuzzy number.

Different Measures of Fuzzy Number:

Let $\widetilde{A} = (a_1, a_2, a_3, a_4)$ be a trapezoidal fuzzy number where $a_1 \leq a_2 \leq a_3 \leq a_4$. Then

Mean $(\overline{X}_{\widetilde{A}}) = \frac{1}{4}(a_1 + a_2 + a_3 + a_4)$

Spread $(S_{\widetilde{A}}) = (a_4 - a_1)$

Area$(A_{\widetilde{A}}) = \frac{1}{2}(a_4 + a_3 - a_1 - a_2)$

If $\widetilde{A} = (a_1, a_2, a_3)$ be a triangle fuzzy number where $a_1 \leq a_2 \leq a_3$. Then

Mean ($\overline{X}_{\widetilde{A}}) = \frac{1}{3}(a_1 + a_2 + a_3)$

Spread ($S_{\widetilde{A}}) = (a_3 - a_1)$

Area$(A_{\widetilde{A}}) = \frac{1}{2}(a_3 - a_1)$

The order of any two real number are easily find out. As example we consider two real number 2 and 5. For these two number we have $2 < 5$ i.e 5 is grater than 2. But for two fuzzy number we cannot easily conclude that which number is maximum. There are several methods are available for ordering the fuzzy number. To compete the order of two fuzzy numbers, we introduced a rank value for each fuzzy number.

Example-1.1: Let us consider two real numbers are represented as triangular fuzzy number as $\widetilde{A} = (2,2,2)$ and $\widetilde{B} = (5,5,5)$. The mean of $\widetilde{A}$ is 2 and that of $\widetilde{B}$ is 5. Hence $\widetilde{A} < \widetilde{B}$. So from above discussion it is seen that the rank value is proportional to the mean position when spreads are constant.

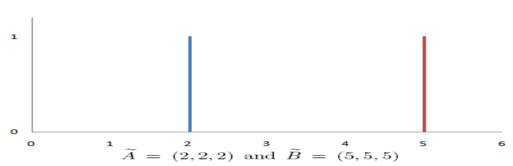

Figure 3.

Example-1.2: If two fuzzy numbers have different spread but mean positions are same, for example, the triangular fuzzy numbers $\widetilde{A} = (1,2,3)$ and $\widetilde{B} = (0,2,4)$. Where mean positions of two fuzzy number are both equal to 2. Then $A_\alpha = [1+\alpha, 3-\alpha]$ and $B_\alpha = [2\alpha, 4-2\alpha]$ where $0 \leq \alpha \leq 1$. Here $\widetilde{A}$ lies in the interval $[1,3]$ and $\widetilde{B}$ lies in the interval $[0,4]$. Every points of the interval $[1,3]$ are the points of $[0,4]$. And hence $[1,3] \subset [0,4]$. i.e.$\widetilde{A} < \widetilde{B}$. Therefore rank value proportionally depends on the spread of the fuzzy numbers when mean position are same.

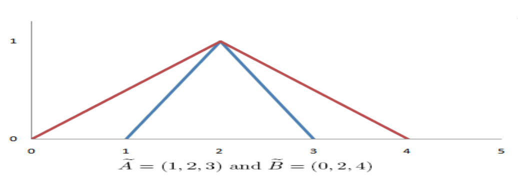

Figure 4.

Example-1.3: Let us consider two fuzzy numbers having different spread and mean positions such as $\widetilde{A} = (1,2,3)$ and $\widetilde{B} = (5,7,10)$. Since spread and mean positions are both vary together, then we cannot apply the above two case. Hence in this case the ordering is proportional to (mean position + spread), therefore there is an individual effect of the mean position and spread in ordering of the fuzzy numbers. Use the above formula the order is $\widetilde{A} < \widetilde{B}$.

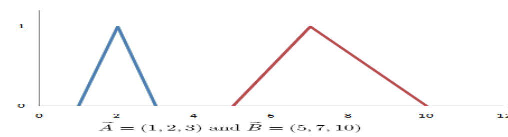

Figure 5.

Example-1.4: Again, we consider two fuzzy numbers with same mean positions and spreads. For example, $\widetilde{A} = (2,5,8)$ and $\widetilde{B} = (2,3,7,8)$. The mean positions and spread of two fuzzy numbers are 5 and 6 respt. According to the Case 3 two fuzzy numbers are same but we see in geometric figure they are not same. Also with respect to area of two fuzzy numbers are not equal and hence the order becomes $\widetilde{A} < \widetilde{B}$. Therefore the rank value is proportional to the area of fuzzy numbers when the mean positions and spread are constant.

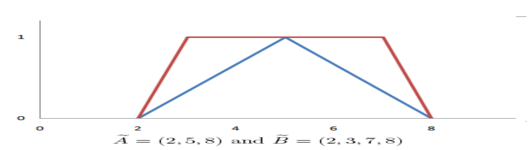

Figure 6.

Rank of a fuzzy number: Use the ideas of upper cases we introduce a new ranking formula for fuzzy numbers. Let us consider a trapezoidal fuzzy number $\widetilde{A} = (a_1, a_2, a_3, a_4)$. Ranking of $\widetilde{A}$ is calculate by following algorithm :

Step-1 : Calculate the mean position $(\overline{X}_{\widetilde{A}})$of the fuzzy number along x-axis.

Step-2 : Calculate the spread $(S_{\widetilde{A}})$of the fuzzy number.

Step-3 : Calculate the area $(A_{\widetilde{A}})$of the fuzzy number.

Step-4 : Calculate the ranking value $(R_{\widetilde{A}})$of the fuzzy number by the following formula.

$$R_{\widetilde{A}} = \frac{1}{3}\{(2 \times \overline{X}_{\widetilde{A}} + S_{\widetilde{A}}) + A_{\widetilde{A}}\} \tag{1}$$

Property -1 : The rank value of any fuzzy number $(a_1, a_2, a_3, a_4)$is less or equal to a_4

Proof : Let $\widetilde{A} = (a_1, a_2, a_3, a_4)$ be a trapezoidal fuzzy number where $a_1 \leq a_2 \leq a_3 \leq a_4$. Then from the above definitions: The ranking value

$$\begin{aligned} R_{\widetilde{A}} &= \frac{1}{3}\{(2\times \overline{X}_{\widetilde{A}} + S_{\widetilde{A}}) + A_{\widetilde{A}}\} \\ &= \frac{1}{3}\{2\times \frac{1}{4}(a_1+a_2+a_3+a_4) + (a_4-a_1) + \frac{1}{2}(a_4+a_3-a_1-a_2)\} \\ &= \frac{1}{3}\{\frac{1}{2}(a_1+a_2+a_3+a_4) + (a_4-a_1) + \frac{1}{2}(a_4+a_3-a_1-a_2)\} \\ &= \frac{1}{3}\{2*(a_4+a_3-a_1)\} \\ &\leq \frac{1}{3}(3*a_4) \qquad [a_3-a_1 \leq a_4-a_1 \leq a_4] \\ &\leq a_4 \end{aligned} \tag{2}$$

Property -2 : The rank value of any fuzzy number $(a_1,a_2,a_3,a_4)$is positive when a_1,a_2,a_3,a_4 are all positive
Proof : Let $\widetilde{A} = (a_1,a_2,a_3,a_4)$ be a trapezoidal fuzzy number where $a_1 \leq a_2 \leq a_3 \leq a_4$. Then the ranking value

$$\begin{aligned} R_{\widetilde{A}} &= \frac{1}{3}\{(2\times \overline{X}_{\widetilde{A}} + S_{\widetilde{A}}) + A_{\widetilde{A}}\} \\ &= \frac{1}{3}\{2\times \frac{1}{4}(a_1+a_2+a_3+a_4) + (a_4-a_1) + \frac{1}{2}(a_4+a_3-a_1-a_2)\} \\ &= \frac{1}{3}\{\frac{1}{2}(a_1+a_2+a_3+a_4) + (a_4-a_1) + \frac{1}{2}(a_4+a_3-a_1-a_2)\} \\ &= \frac{1}{3}(2*a_4+a_3-a_1) \\ &\geq \frac{1}{3}(2*a_4) \qquad [a_3-a_1 \geq 0] \\ &> 0 \qquad [a_4 > 0] \end{aligned} \tag{3}$$

Example-2.1: The order relations of three fuzzy numbers $\widetilde{A}_1 = (0.4,0.5,1)$, $\widetilde{A}_2 = (0.4,0.7;1)$, $\widetilde{A}_3 = (0.4,0.9,1)$, is represented in Figure 7.The rank value of three fuzzy numbers are $R_{\widetilde{A_1}} = 0.722$, $R_{\widetilde{A_2}} = 0.767$, $R_{\widetilde{A_3}} = 0.811$ (Followed 1).Hence $\widetilde{A_1} < \widetilde{A_2} < \widetilde{A_3}$.

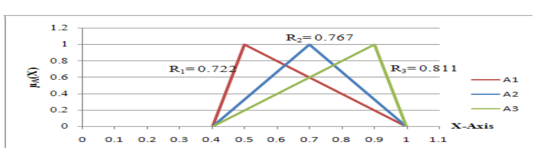

Figure 7.

Example-2.2: The two fuzzy numbers in (represent in Figure 8)are respectively $\widetilde{A}_1(0.2,0.5,0.8)$, $\widetilde{A}_2(0.4,0.5;0.6)$, then we have: $R_{\widetilde{A_1}} = 0.633$, $R_{\widetilde{A_2}} = 0.433$. Hence $\widetilde{A_1} > \widetilde{A_2}$.
Example-2.3: The three fuzzy numbers in (represent in Figure 9 are respectively
$\widetilde{A}_1(0.5,0.7,0.9)$, $\widetilde{A}_2(0.3,0.7,0.9)$, $\widetilde{A}_3(0.3,0.4,0.7,0.9)$, then

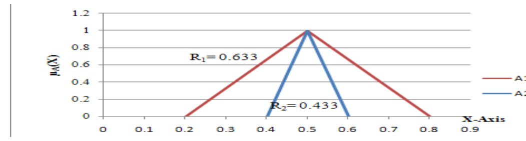

Figure 8.

we have: $R_{\widetilde{A_1}} = 0.667$, $R_{\widetilde{A_2}} = 0.722$, $R_{\widetilde{A_3}} = 0.733$. Hence $\widetilde{A_1} < \widetilde{A_2} < \widetilde{A_3}$.

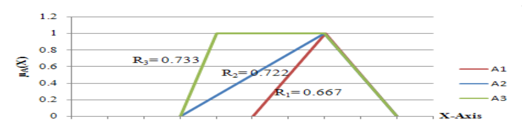

Figure 9.

Example-2.4: The three fuzzy numbers in (represent in Figure 10) are respectively
$\widetilde{A}_1(0.3,0.5,0.8,0.9)$, $\widetilde{A}_2(0.3,0.5,0.9)$, $\widetilde{A}_3(0.3,0.5,0.7)$, then we have: $R_{\widetilde{A_1}} = 0.767$, $R_{\widetilde{A_2}} = 0.678$, $R_{\widetilde{A_3}} = 0.533$, hence $\widetilde{A_3} < \widetilde{A_2} < \widetilde{A_1}$.

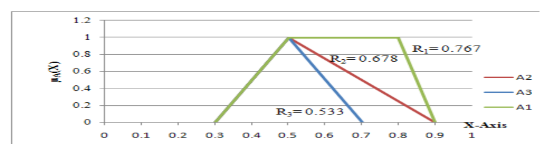

Figure 10.

Example-2.5: The two fuzzy numbers in (represent in Figure 11:) are respectively $\widetilde{A}_1(0.3,0.3,1)$, $\widetilde{A}_2(0.1,0.7,0.8)$, then we have: $R_{\widetilde{A_1}} = 0.706$, $R_{\widetilde{A_2}} = 0.706$, hence $\widetilde{A_1} = \widetilde{A_2}$.

III. NOTATIONS AND ASSUMPTIONS OF THE PROPOSED MODEL

A. Notations and Assumptions

In this solid transportation problem the following notations have been used

(i) M = no of sources of the transportation problem.

(ii) N = no of demands of the transportation problem.

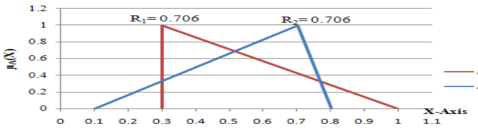

Figure 11.

(iii) K = no of conveyances i.e., different modes of the transportation problem.

(iv) O_i= origins of the transportation problem.

(v) D_j= destination of the transportation problem.

(vi) E_k=conveyances of the transportation problem.

(vii) a_i= amount of a homogeneous product available at i-th origin.

(viii) b_j=demand at j-th destination.

(viii) e_k=amount of product which can be carried by k-th conveyance.

(ix) $C_{ijk}, \widetilde{C}_{ijk}$ = crisp and fuzzy[triangular, of the form $(C^L_{ijk}, C^C_{ijk}, C^U_{ijk})$] unit transportation cost.

(x) x_{ijk} = amount of the product to be transported from i-th supply to j-th demand node by k-th conveyance.

IV. FORMULATION OF FUZZY SOLID TRANSPORTATION MODEL(FSTM)

In this paper, a transportation problem is considered with M supply nodes, N demand nodes and K conveyances, in which a_i units are supplied by i-th supply node, b_j units are required by j-th demand node and e_j units are carried by k-th conveyance. There is a unit shipping cost, $\widetilde{c}_{ijk}$ for transportation from i-th supply node to j-th demand node by k-th conveyance. Let x_{ijk} denote the number of units to be transported from i-th supply to j-th destination by k-th conveyance respectively. Hence the proposed solid transportation problem in fuzzy environment has been considered as:

$$\text{Minimize } \widetilde{Z} = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} \widetilde{c}_{ijk}\, x_{ijk} \tag{4}$$

subject to constraints

$$\begin{aligned}
&\sum_{j=1}^{N}\sum_{k=1}^{K} x_{ijk} \leq a_i, && i = 1,2,......M\\
&\sum_{i=1}^{M}\sum_{k=1}^{K} x_{ijk} \geq b_j, && j = 1,2......N\\
&\sum_{i=1}^{M}\sum_{j=1}^{N} x_{ijk} \leq e_k, && k = 1,2,......K\\
&\sum_{i=1}^{M} a_i = \sum_{j=1}^{N} b_j = \sum_{k=1}^{K} e_j\\
&\text{and } x_{ijk} \geq 0, && \forall i,j,k.
\end{aligned} \tag{5}$$

The objective function $\widetilde{Z}$ presents in equation(4) representing the total transportation cost. The first three constraints present in equation(5) represents the availability constraints, demand constraints and capacity constraints of the origins, destination and conveyance respectively.The fourth constraints represents the balance criteria and last constraints represents the feasibility condition of the STP.

V. ALGORITHM FOR PROPOSED METHOD

Step 1: Check the given STP is balanced. If not, change into a balance one by introducing one dummy source or destination or conveyance having zero transportation cost.

Step 2: Identify the boxes having minimum and next to minimum transportation cost in each source and write their difference (penalty)using equation(1) , along the side of the table against the corresponding source.(Identify the boxes having minimum and next to minimum transportation cost is done by comparison the rank of transportation cost)

Step 3: We apply the similar process step-2 for demands and conveyance

Step 4: Identify the maximum penalty (through compairing the rank) from the side of the table, make maximum allotment to the box having minimum cost of transportation in that source or demand or conveyance. If in the table, two or more penalties are equal, you are at liberty to break the tie arbitrarily.

Step 5: Discard the fulfilment source or destination or conveyance (followed by step 4) and repeat the above steps (2-4) in the remaining problem until all restrictions are satisfied.

Step 6: The number of B.F.S. (obtained by followed steps 2-5) are $(M+N+K-2)$. If one of basic variable in the optimal solution of the problem is zero, that is, the number of nonzero basic variables is less than $(M+N+K-2)$, then the problem is called degenerate, then followed step-7; Otherwise it is called non-degenerate.and goto step-8

Step 7: For degenerate B.F.S, we assign a small positive quantity ε in a non allocated cell and goto step-8.

Step 8: Determine the set of $(M+N+K)$ numbers MODI-indices $u_i(i=1,2,\ldots.M)$,$v_j(j=1,2,\ldots N)$ and $w_k(j=1,2,\ldots K)$. By taking two MODI-indices values zero and applying the conditions $\widetilde{c}_{ijk}=\widetilde{u}_i+\widetilde{v}_j+\widetilde{w}_k$, for all basic cells (i,j,k), we can compute the rest of the MODI-indices.

Step 9: For non basic cell (i,j,k) we compute $\widetilde{d}_{ijk}$ by using the formula $\widetilde{d}_{ijk}=\widetilde{c}_{ijk}-(\widetilde{u}_i+\widetilde{v}_j+\widetilde{w}_k)$

Step 10: Examine the matrix of cell evaluation $\widetilde{d}_{ijk}$ for negative entries and conclude that

(i) If all $\widetilde{d}_{ijk}>\widetilde{0}\Rightarrow$ Solution is optimal and unique.

(ii)If all $\widetilde{d}_{ijk}\geq\widetilde{0}$ and at least one $\widetilde{d}_{ijk}\sim\widetilde{0}$

$\Rightarrow$ Solution is optimal and alternate solution also exists.

(iii) If at least one $\widetilde{d}_{ijk}<\widetilde{0}$?Solution is not optimal. If it is so, further improvement is required by following step - 11 onwards.

Step 11:

(i) See the most negative cell in the matrix [$\widetilde{d}_{ijk}$].

(ii) Allocate θ to this empty cell in the final allocation table. Subtract and add the amount of this allocation to other corners of the loop in order to restore feasibility.

(iii) The value of θ, in general is obtained by equating to zero the minimum of the allocations containing θ

(iv)Substitute the value of θ and find a fresh allocation table.

Step 12: Again, apply the above test for optimality till you find all $\widetilde{d}_{ijk}\geq\widetilde{0}$

VI. NUMERICAL ILLUSTRATION

Consider a solid transportation problem with three suppliers, three destinations and three conveyances in fuzzy environment with their shipping cost as given in Table 1.

Step-2: To compare the given fuzzy cost, rank of each costs are calculate followed definition in Table 2.

Where R denote Rank of respective fuzzy number.

Step-3: Following step-3, choose two minimum fuzzy cost for each origin, destination and conveyance than calculate the corresponding penalties.

For example, Along E_1, two minimum fuzzy costs are (0,1,2) and (3,4,5) (since their ranks are minimum) and corresponding penalty is (1,3,5).

Similarly other penalties are, for E_2: (-1,1,3)=(1,2,3)-(0,1,2)

for E_3: (-2,-1,0)=(1,2,3)-(3,3,3)

for O_1: (-2,0,6)=(1,2,7)-(1,2,3)

for O_2: (-1,1,3)=(1,2,3)-(0,1,2)

for O_3:(1,2,3)=(3,3,3)-(0,1,2)

for D_1: (-1,1,3)=(1,2,3)-(0,1,2)

for D_2: (1,2,3)=(3,3,3)-(0,1,2)

for D_3: (0,2,4)=(3,4,5)-(1,2,3)

Next, select the maximum penalties (followed rank method) (-2,0,6) corresponds to O_1 and followed step-5, allocate maximum possible amount to the cell (1,3,3). And proceeding in this way, we get all the allocation as in Table 3.

The number of B.F.S. is 7 =(M+N+L-2), so the solution is non-degenerate. Then, we test the optimality by computing the MODI indices $\widetilde{u}_i,\widetilde{v}_j$ and $\widetilde{w}_k$ such that for for all basic cells (i,j,k), $\widetilde{c}_{ijk}=\widetilde{u}_i+\widetilde{v}_j+\widetilde{w}_k$, and for non basic cell (i,j,k) we compute $\widetilde{d}_{ijk}$ (such fuzzy numbers are enclosed in [ ])by using the formula $\widetilde{d}_{ijk}=\widetilde{c}_{ijk}?(\widetilde{u}_i+\widetilde{v}_j+\widetilde{w}_k)$ we get table 4.

Since all $\widetilde{d}_{ijk}$ are positive . Therefore this is the optimal table and optimal solution as $x_{132}=2$, ,$x_{133}=9$,$x_{221}=11$, $x_{222}=2$,$x_{312}=7$,$x_{322}=2$, $x_{332}=1$ and the corresponding $\widetilde{Z}$=(30,74,121)

To compaire the results obtained through modified VAM using ranking method, we consider the same imprecise STP. In this method using 'extension principle', we convert the objective function into a weighted average function

$$\widetilde{Z}(x_{ijk}) = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\left(\frac{C_{ijk}^{L}+2C_{ijk}^{C}+C_{ijk}^{R}}{4}\right) x_{ijk}$$

and minimize $\widetilde{Z}(x_{ijk})$ using Lingo-9.0 software. The obtained results are shown in table 5.

The corresponding value of =75.25, which shown that the results obtained through modified VAM by using ranking method is better than the results obtained through Lingo-9.0 software by using extension principle and weighted average value.

VII. SENSITIVITY ANALYSIS

A. Bound technique

Proposition-1: Let (i,j,k)th cell be a non-basic cell corresponding to an optimal solution of the STP with $d_{ijk} = \widetilde{c}_{ijk} - \widetilde{u}_i - \widetilde{v}_j - \widetilde{w}_k (\geq 0)$. If $\widetilde{c}_{ijk} + \widetilde{\triangle}_{ijk}$ is the perturbed cost of $\widetilde{c}_{ijk}$, then the range of $\widetilde{c}_{ijk} = [\widetilde{\triangle}_{ijk}, \infty)$

Proof: Now, since (i,j,k)th cell is a non-basic cell and the perturbed cost $\widetilde{c}_{ijk} + \widetilde{\triangle}_{ijk}$ is not affected the current optimal solution to the problem, $\widetilde{c}_{ijk} - \widetilde{u}_i - \widetilde{v}_j - \widetilde{w}_k (\geq 0)$. This implies that,$\widetilde{d}_{ijk} \leq -\widetilde{\triangle}_{ijk}$ . . Therefore, the range of $\widetilde{\triangle}_{ijk} = [-\widetilde{d}_{ijk}, \infty)$ Hence the theorem.

Proposition-2: Let (i,j,k)th cell be basic cell corresponding to an optimal solution of the STP with $\widetilde{d}_{ijk} = \widetilde{c}_{ijk} - \widetilde{u}_i - \widetilde{v}_j - \widetilde{w}_k (= 0)$. If $\widetilde{c}_{ijk} + \widetilde{\triangle}_{ijk}$ is the perturbed cost of $\widetilde{c}_{ijk}$ and $\widetilde{U}i$ is the minimum value of $\widetilde{d}_{ijk}$ for all non-basic cells in the ith origin, $\widetilde{V}_j$ is the minimum value of $\widetilde{d}_{ijk}$ for all non-basic cells in the jth destination and $\widetilde{W}_k$ is the minimum value of $\widetilde{d}_{ijk}$ for all non-basic cells in the kth conveyance, then the range of $\widetilde{\triangle}_{ijk} = (-\infty, \widetilde{M}_{ijk}]$ where $\widetilde{M}_{ijk}$ = the maximum $\{\widetilde{U}_i, \widetilde{V}_j, \widetilde{W}_k\}$; **Proof:** Now, since $\widetilde{c}_{ijk} + \widetilde{\triangle}_{ijk}$ is the perturbed value of $\widetilde{c}_{ijk}$ and the current optimal solution remains optimal, $\widetilde{d}_{ijk} = \widetilde{c}_{ijk} - \widetilde{u}_i - \widetilde{v}_j - \widetilde{w}_k \geq \widetilde{0}$, for all non-basic cells in the ith origin , the jth destination and the kth conveyance are positive. Now, attaching the $\widetilde{\triangle}_{ijk}$ to first u_i , then v_j and then w_k , we have the following:
$\widetilde{c}_{imn} - (\widetilde{u}_i + \widetilde{\triangle}_{ijk}) - \widetilde{v}_m - \widetilde{w}_n \geq \widetilde{0}$,(i,m,n) is non - basic cells, for all m and n ;
$\widetilde{c}_{pjn} - \widetilde{u}_i - (\widetilde{v}_m + \widetilde{\triangle}_{ijk}) - \widetilde{w}_n \geq \widetilde{0}$, (p, j,n) is non - basic cells, for all p and n and
$\widetilde{c}_{pmk} - \widetilde{u}_i - \widetilde{v}_m - (\widetilde{w}_n + \widetilde{\triangle}_{ijk}) \geq \widetilde{0}$, (p,m,k) is non - basic cells, for all p and m . Then, we can conclude that is implies that $\widetilde{\triangle}_{ijk} \leq \widetilde{U}_i$;$\widetilde{\triangle}_{ijk} \leq \widetilde{V}_j$ and $\widetilde{\triangle}_{ijk} \leq \widetilde{W}_k$. Now, since we attach any one of the MODI-indices $\widetilde{u}_i$,$\widetilde{v}_j$ and $\widetilde{w}_k$, we take, $\widetilde{M}_{ijk}$ = maximum $\widetilde{U}_i, \widetilde{V}_j, \widetilde{W}_k$ for getting better range. Therefore, the range of $\widetilde{\triangle}_{ijk} = (-\infty, \widetilde{M}_{ijk}]$. Hence the theorem.

B. Effect of change in the components of sources, destinations and conveyances

Any change of the source component or destination component or conveyance component, does not affect the optimality condition i.e, the allocation co-ordinate does not change with the change of availability, resource or capacity of conveyance. But it changes with the amount of transportation. Here percentage of affect in total cost due to the change of a_i, b_j, e_k in $\widetilde{Z}$ are given in Table 6.

VIII. CONCLUDING REMARKS

Here, for the first time, the Vogel Approximation method has been introduced in a three-dimensional transportation problem. Here, the unit transportation costs are assumed imprecise in nature and their comparison, algebraic operations are performed by using the concept of rank of fuzzy number. The obtained solution is globally optimized, which is self explained. A sensitive analysis has been done for the availabilities, demands and capacities of the considerable illustration (shown in table- 6), which shows the effects of such parameters on the total transportation cost. Others transportation solution technique, like North West corner method, matrix minimum method also can be applied to find the basic feasible solution of such solid transportation problem. Degeneracy problem also can be solved in this regards, which can be considered in near future. This paper also can be extended for multi-criteria decision making problem.

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Table-1:

	D_1			D_2			D_3			Capacity
	E_1	E_2	E_3	E_1	E_2	E_3	E_1	E_2	E_3	
O_1	(1,3,9)	(2,8,10)	(3,9,11)	(1,2,7)	(6,8,14)	(3,8,9)	(1,5,9)	(2,6,11)	(1,2,3)	11
O_2	(3,4,5)	(1,2,3)	(0,6,8)	(0,1,2)	(1,3,5)	(6,8,10)	(2,7,16)	(3,3,7)	(3,6,9)	13
O_3	(3,9,11)	(0,1,2)	(2,3,4)	(2,4,6)	(5,7,9)	(3,3,3)	(4,5,7)	(5,6,8)	(3,4,5)	10
Conveyance	11	14	9	11	14	9	11	14	9	
Demand	7			15			12			

Table-2:

	D_1			D_2			D_3			Capacity
	E_1	E_2	E_3	E_1	E_2	E_3	E_1	E_2	E_3	
O_1	(1,3,9) R=6.89	(2,8,10) R=8.44	(3,9,11) R=9.11	(1,2,7) R=5.22	(6,8,14) R=10.22	(3,8,9) R=7.44	(1,5,9) R=7.33	(2,6,11) R=8.72	(1,2,3) R=2.33	11
O_2	(3,4,5) R=3.67	(1,2,3) R=2.33	(0,6,8) R=7.11	(0,1,2) R=1.67	(1,3,5) R=4	(6,8,10) R=7.33	(2,7,16) R=12.56	(3,3,7) R=4.89	(3,6,9) R=7	13
O_3	(3,9,11) R=9.11	(0,1,2) R=1.67	(2,3,4) R=3	(2,4,6) R=4.66	(5,7,9) R=6.67	(3,3,3) R=2	(4,5,7) R=5.05	(5,6,8) R=5.72	(3,4,5) R=4.44	10
Conveyance	11	14	9	11	14	9	11	14	9	
Demand	7			15			12			

Table-3:

	D_1			D_2			D_3			Capacity
	E_1	E_2	E_3	E_1	E_2	E_3	E_1	E_2	E_3	
O_1	0 (1,3,9)	0 (2,8,10)	0 (3,9,11)	0 (1,2,7)	0 (6,8,14)	0 (3,8,9)	0 (1,5,9)	2 (2,6,11)	9 (1,2,3)	11
O_2	0 (3,4,5)	0 (1,2,3)	0 (0,6,8)	11 (0,1,2)	2 (1,3,5)	0 (6,8,10)	0 (2,7,16)	0 (3,3,7)	0 (3,6,9)	13
O_3	0 (3,9,11)	7 (0,1,2)	0 (2,3,4)	0 (2,4,6)	2 (5,7,9)	0 (3,3,3)	0 (4,5,7)	1 (5,6,8)	0 (3,4,5)	10
Conveyance	11	14	9	11	14	9	11	14	9	
Demand	7			15			12			

Table-4:

	D_1			D_2			D_3			u_i
	E_1	E_2	E_3	E_1	E_2	E_3	E_1	E_2	E_3	
O_1	[-16,4,28] (1,3,9)	[-14,7,24] (2,8,10)	[-21,12,42] (3,9,11)	[-19,-3,17] (1,2,7)	[-13,1,19] (6,8,14)	[-24,5,31] (3,8,9)	[-22,1,23] (1,5,9)	(-20,0,20) (2,6,11)	(-29,0,29) (1,2,3)	(-10,0,10)
O_2	[-4,9,22] (3,4,5)	[-5,5,15] (1,2,3)	[-14,13,37] (0,6,8)	(-10,0,10) (0,1,2)	(-8,0,8) (1,3,5)	[-11,9,30] (6,8,10)	[-11,7,28] (2,7,16)	[-9,1,14] (3,3,7)	[-17,8,33] (3,6,9)	(-8,-4,0)
O_3	[-4,10,20] (3,9,11)	(-6,0,6) (0,1,2)	[-12,6,25] (2,3,4)	[-8,-1,6] (2,4,6)	(-4,0,4) (5,7,9)	[-14,0,15] (3,3,3)	[-9,1,11] (4,5,7)	(-7,0,7) (5,6,8)	[-19,3,21] (3,4,5)	(0,0,0)
w_k	$(0,5,10)$	$(5,7,9)$	$(-12,3,17)$	$(0,5,10)$	$(5,7,9)$	$(-12,3,17)$	$(0,5,10)$	$(5,7,9)$	$(-12,3,17)$	
v_j	$(-9,-6,-3)$			$(0,0,0)$			$(-4,-1,3)$			

Table-5:

	D_1			D_2			D_3			Capacity
	E_1	E_2	E_3	E_1	E_2	E_3	E_1	E_2	E_3	
O_1	0 (1,3,9)	0 (2,8,10)	0 (3,9,11)	2 (1,2,7)	0 (6,8,14)	0 (3,8,9)	0 (1,5,9)	0 (2,6,11)	9 (1,2,3)	11
O_2	0 (3,4,5)	0 (1,2,3)	0 (0,6,8)	6 (0,1,2)	4 (1,3,5)	0 (6,8,10)	0 (2,7,16)	3 (3,3,7)	0 (3,6,9)	13
O_3	0 (3,9,11)	7 (0,1,2)	0 (2,3,4)	3 (2,4,6)	0 (5,7,9)	0 (3,3,3)	0 (4,5,7)	0 (5,6,8)	0 (3,4,5)	10
Conveyance	11	14	9	11	14	9	11	14	9	
Demand	7			15			12			

Table-6: Sensitivity Analysis

a_1(%)	a_2(%)	a_3(%)	b_1(%)	b_2(%)	b_3(%)	e_1(%)	e_2(%)	e_3(%)	Z(%)
-40	-40	-40	-40	-40	-40	-40	-40	-40	-40
$-18\cdot 18$	$-15\cdot 38$	$-20\cdot 00$	$-28\cdot 57$	$-13\cdot 33$	$-16\cdot 66$	$-18\cdot 18$	$-24\cdot 29$	$-22\cdot 22$	$-11\cdot 43$
$-9\cdot 09$	$-7\cdot 69$	$-10\cdot 00$	$-14\cdot 3$	$-6\cdot 67$	$-8\cdot 33$	$-9\cdot 09$	$-7\cdot 14$	$-11\cdot 1$	$-5\cdot 71$
-5	-5	-5	-5	-5	-5	-5	-5	-5	-5
0	0	0	0	0	0	0	0	0	0
+5	+5	+5	+5	+5	+5	+5	+5	+5	+5
$+9\cdot 09$	$+7\cdot 69$	$+10\cdot 00$	$+14\cdot 3$	$+6\cdot 67$	$+8\cdot 33$	$+9\cdot 09$	$+7\cdot 14$	$+11\cdot 1$	$+5\cdot 71$
$+18\cdot 18$	$+15\cdot 38$	$+20\cdot 00$	$+28\cdot 57$	$+13\cdot 33$	$+16\cdot 66$	$+18\cdot 18$	$+24\cdot 29$	$+22\cdot 22$	$+11\cdot 43$
+40	+40	+40	+40	+40	+40	+40	+40	+40	+40